

# Casimir effect for thin films in QED

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**Abstract.** We consider the problem of modeling of interaction of thin material films with fields of quantum electrodynamics. Taking into account the basic principles of quantum electrodynamics (locality, gauge invariance, renormalizability) we construct a single model for Casimir-like phenomena arising near the film boundary on distances much larger than Compton wavelength of the electron. In this region contribution of Dirac fields fluctuations are not essential and can be neglected. In the model the film is presented by a singular background field concentrated on a 2-dimensional surface and interacting with quantum electromagnetic field. All properties of the film material are described by one dimensionless parameter. For two parallel plane films the Casimir force appears to be non-universal and dependent on material property. It can be both attractive and repulsive. In the model we study scattering of electromagnetic wave on the plane film, an interaction of plane film with point charge, homogeneously charged plane and straight line current. Here, besides usual results of classical electrodynamics the model predicts appearance of anomalous electromagnetic phenomena.

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## 1. Introduction

In 1948 it was shown by Casimir that vacuum fluctuations of quantum fields generate an attraction between two parallel uncharged conducting planes [1]. This phenomena called the Casimir effect (CE) has been well investigated with methods of modern experiments [2, 3, 4]. The CE is a manifestation of influence of fluctuations of quantum fields on the level of classical interaction of material objects. Theoretical and experimental investigation of phenomena such a kind became very important for development of micro-mechanics and nano-technology.

Though there are many theoretical results on the CE [5], however the majority of them are received in framework of several models based not on the quantum electrodynamics (QED) directly. Usually, one assumes that the CE can be investigated in the framework of free massless quantum scalar field theory with fixed boundary conditions or  $\delta$ -function potentials [6, 7] ignoring restrictions following from gauge invariance, locality and renormalizability of QED. By means of such methods one can investigate some of the CE properties, but there is no possibility of studying other phenomena generated by interaction of the QED fields with considered classical background within the same model.

An approach for construction of the single QED model for investigation of all peculiar properties of the CE for thin material films was proposed in [8, 9]. We consider its application for simple case of parallel plane films. We show that gauge invariance, locality and renormalizability considered as basic principles make strong restrictions for constructions of the CE models in QED, which make it possible to reveal new important features of the CE-like phenomena.

## 2. Construction of models

We construct models for interaction of the material film with QED fields on the basis of most general assumptions. We suppose that the film is presented by a singular background (defect) concentrated on the 2-dimensional surface. Its interaction with QED fields has most general form defined by the geometry of the defect and restrictions following from the basic principles of QED (gauge invariance, locality, renormalizability). The locality of interaction means that the action functional of the defect is represented by an integral over defect surface of the Lagrangian density which is a polynomial function of space-time point in respect to fields and derivatives of ones. The coefficients of this polynomial are the parameters defining defect properties. For the quantum field theory (QFT) with singular background the requirement of renormalizability was analyzed by Symanzik in [10]. He showed that in order to keep renormalizability of the model, one needs to add a defect action to the usual bulk action of QFT model. The defect action must contain all possible terms with nonnegative dimensions of parameters and not include any parameters with negative dimensions. In case of QED the defect action must be also gauge invariant.

From these requirements it follows that for thin film (without charges and currents) which shape is defined by equation  $\Phi(x) = 0$ ,  $x = (x_0, x_1, x_2, x_3)$ , the action describing its interaction with photon field  $A_\mu(x)$  reads

$$S_\Phi(A) = \frac{a}{2} \int \varepsilon^{\lambda\mu\nu\rho} \partial_\lambda \Phi(x) A_\mu(x) F_{\nu\rho}(x) \delta(\Phi(x)) dx \quad (1)$$

where  $F_{\nu\rho}(x) = \partial_\nu A_\rho - \partial_\rho A_\nu$ ,  $\varepsilon^{\lambda\mu\nu\rho}$  denotes totally antisymmetric tensor ( $\varepsilon^{0123} = 1$ ),  $a$  is a constant dimensionless parameter. The action (1) is a surface Chern-Simon action [11, 12]. The fermion defect action can be written as

$$S_\Phi(\bar{\psi}, \psi) = \int \bar{\psi}(x) [\lambda + u^\mu \gamma_\mu + \gamma_5 (\tau + v^\mu \gamma_\mu) + \omega^{\mu\nu} \sigma_{\mu\nu}] \psi(x) \delta(\Phi(x)) dx \quad (2)$$

Here,  $\gamma_\mu$ ,  $\mu = 0, 1, 2, 3$ , are the Dirac matrices,  $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ ,  $\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$ , and  $\lambda, \tau, u_\mu, v_\mu, \omega^{\mu\nu} = -\omega^{\nu\mu}$ ,  $\mu, \nu = 0, 1, 2, 3$  are 16 dimensionless parameters.

Expressions (1), (2) are the most general forms of gauge invariant actions concentrated on the defect surface being invariant in respect to reparametrization of one and not having any parameters with negative dimensions.

We consider in this paper CE-like phenomena arising on the distances from the defect boundary much larger then Compton wavelength of the electron. In this case one can neglect the Dirac fields in QED because of exponential damping of fluctuations of those on much smaller distances ( $\sim m_e^{-1} \approx 10^{-10} cm$  for electron,  $\sim m_p^{-1} \approx 10^{-13} cm$  for proton [8]). Thus, for constructing of model we can use the action of free quantum electromagnetic field (photodynamic) with additional defect action (1).

For description of all physical phenomena it is enough to calculate generating functional of Greens functions. For gauge condition  $\phi(A) = 0$  it reads

$$G(J) = C \int e^{iS(A, \Phi) + iJA} \delta(\phi(A)) DA \quad (3)$$

where

$$S(A, \Phi) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + S_\Phi(A), \quad (4)$$

and the constant  $C$  is defined by normalization condition  $G(0)|_{a=0} = 1$ . The first term on the right hand side of (4) is the usual action of photon field. Along with defect action it forms a quadratic in photon field full action of the system which can be written as  $S(A, \Phi) = 1/2 A_\mu K_\Phi^{\mu\nu} A_\nu$ . The integral (3) is gaussian and is calculated exactly:

$$G(J) = \exp \left\{ \frac{1}{2} Tr \ln(D_\Phi D^{-1}) - \frac{1}{2} J D_\phi J \right\}$$

where  $D_\Phi$  is the propagator  $D_\Phi = iK_\Phi^{-1}$  of photodynamic with defect in gauge  $\phi(A) = 0$ , and  $D$  is the propagator of photon field without defect in the same gauge. For the static defect, function  $\Phi(x)$  is time independent, and  $\ln G(0)$  defines the Casimir energy.

In order to expose essential Feature of CE-like phenomena in constructed model, we calculate the Casimir force (CF) for simple case of two parallel infinite plane films and study a scattering of electromagnetic wave on the plane defect. We consider also an interaction of the plane film with a parallel to it straight line current and an interaction of film with a point charge and homogeneous charge distribution on parallel plane.

### 3. Casimir force

We consider defect concentrated on two parallel planes  $x_3 = 0$  and  $x_3 = r$ . For this model, it is convenient to use a notation like  $x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$ . Defect action (1) has the form:

$$S_{2P} = \frac{1}{2} \int (a_1 \delta(x_3) + a_2 \delta(x_3 - r)) \varepsilon^{3\mu\nu\rho} A_\mu(x) F_{\nu\rho}(x) dx.$$

The defect action  $S_{2P}$  was discussed in [13] in substantiation of Chern-Simon type boundary conditions chosen for studies of the Casimir effect in photodynamics. This based on boundary conditions approach is not related directly to the one we present. The defect action (1) is the main point in our model formulation, and no any boundary conditions are used. The action  $S_{2P}$  is translationally invariant with respect to coordinates  $x_i$ ,  $i = 0, 1, 2$ . The propagator  $D_\Phi(x, y)$  is written as:

$$D_{2P}(x, y) = \frac{1}{(2\pi)^3} \int D_{2P}(\vec{k}, x_3, y_3) e^{i\vec{k}(\vec{x}-\vec{y})} d\vec{k},$$

and  $D_{2P}(\vec{k}, x_3, y_3)$  can be calculated exactly. Using latin indexes for the components of 4-tensors with numbers 0, 1, 2 and notations

$$P^{lm}(\vec{k}) = g^{lm} - k^l k^m / \vec{k}^2, \quad L^{lm}(\vec{k}) = \epsilon^{lmn3} k_n / |\vec{k}|, \quad \vec{k}^2 = k_0^2 - k_1^2 - k_2^2, \quad |\vec{k}| = \sqrt{\vec{k}^2}$$

( $g$  is metrics tensor), one can present the results for the Coulomb-like gauge  $\partial_0 A^0 + \partial_1 A^1 + \partial_2 A^2 = 0$  as follows [9]

$$D_{2P}^{33}(\vec{k}, x_3, y_3) = \frac{-i\delta(x_3 - y_3)}{|\vec{k}|^2}, \quad D_{2P}^{l3}(\vec{k}, x_3, y_3) = D_{2P}^{3m}(\vec{k}, x_3, y_3) = 0,$$

$$D_{2P}^{lm}(\vec{k}, x_3, y_3) = \frac{P^{lm}(\vec{k}) \mathcal{P}_1(\vec{k}, x_3, y_3) + L^{lm}(\vec{k}) \mathcal{P}_2(\vec{k}, x_3, y_3)}{2|\vec{k}|[(1 + a_1 a_2 (e^{2i|\vec{k}|\Gamma} - 1))^2 + (a_1 + a_2)^2]}$$

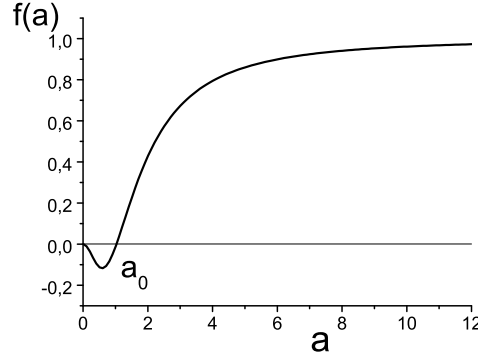
where

$$\begin{aligned} \mathcal{P}_1(\vec{k}, x_3, y_3) &= [a_1 a_2 - a_1^2 a_2^2 (1 - e^{2i|\vec{k}|\Gamma})] [e^{i|\vec{k}|(|x_3|+|y_3-\Gamma|)} + e^{i|\vec{k}|(|x_3-\Gamma|+|y_3|)}] e^{i|\vec{k}|\Gamma} + \\ &+ [a_1^2 + a_1^2 a_2^2 (1 - e^{2i|\vec{k}|\Gamma})] e^{i|\vec{k}|(|x_3|+|y_3|)} + [a_2^2 + a_1^2 a_2^2 (1 - e^{2i|\vec{k}|\Gamma})] e^{i|\vec{k}|(|x_3-\Gamma|+|y_3-\Gamma|)} - \\ &- e^{i|\vec{k}||x_3-y_3|} [(1 + a_1 a_2 (e^{2i|\vec{k}|\Gamma} - 1))^2 + (a_1 + a_2)^2], \\ \mathcal{P}_2(\vec{k}, x_3, y_3) &= a_1 [1 + a_2 (a_2 + a_1 e^{2i|\vec{k}|\Gamma})] e^{i|\vec{k}|(|x_3|+|y_3|)} + \\ &+ a_2 [1 + a_1 (a_1 + a_2 e^{2i|\vec{k}|\Gamma})] e^{i|\vec{k}|(|x_3-\Gamma|+|y_3-\Gamma|)} - \\ &- a_1 a_2 (a_1 + a_2) \left( e^{i|\vec{k}|(|x_3|+|y_3-\Gamma|)} + e^{i|\vec{k}|(|x_3-\Gamma|+|y_3|)} \right) e^{i|\vec{k}|\Gamma}. \end{aligned}$$

The energy density  $E_{2P}$  of defect is defined as

$$\ln G(0) = \frac{1}{2} \text{Tr} \ln(D_{2P} D^{-1}) = -iT S E_{2P}$$

where  $T = \int dx_0$  is duration of defect, and  $S = \int dx_1 dx_2$ , is the area of film. It is expressed in an explicit form in terms of polylogarithm function  $\text{Li}_4(x)$  [9]. For identical



**Figure 1.** Function  $f(a)$  determining Casimir force between parallel planes

films with  $a_1 = a_2 = a$  it holds:  $E_{2P} = 2E_s + E_{Cas}$ ,  $E_s = \int \ln \sqrt{(1 + a^2)} \frac{d\vec{k}}{(2\pi)^3}$ ,

$$E_{Cas} = -\frac{1}{16\pi^2 r^3} \left\{ \text{Li}_4 \left( \frac{a^2}{(a+i)^2} \right) + \text{Li}_4 \left( \frac{a^2}{(a-i)^2} \right) \right\}.$$

Here  $E_s$  is an infinite constant, which can be interpreted as self-energy density on the plane, and  $E_{Cas}$  is an energy density of their interaction. Function  $\text{Li}_4(x)$  is defined as  $\text{Li}_4(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^4} = -\frac{1}{2} \int_0^{\infty} k^2 \ln(1 - xe^{-k}) dk$ . The force  $F_{2P}(r, a)$  between planes is given by

$$F_{2P}(r, a) = -\frac{\partial E_{Cas}(r, a)}{\partial r} = -\frac{\pi^2}{240r^4} f(a).$$

The force  $F_{2P}$  is repulsive for  $|a| < a_0$  and attractive for  $|a| > a_0$ ,  $a_0 \approx 1.03246$  (see Figure 1). For large  $|a|$  it is the same as the usual CF between perfectly conducting planes. The model predicts that the maximal magnitude of the *repulsive*  $F_{2P}$  is expected for  $|a| \approx 0.6$ . For two infinitely thick parallel slabs the repulsive CF was predicted also in [14].

Real film has a finite width, and the bulk contributions to the CF for nonperfectly conducting slabs with widths  $h_1, h_2$  are proportional to  $h_1 h_2$ . Therefore it follows directly from the dimensional analysis that the bulk correction  $F_{bulk}$  to the CF is of the form  $F_{bulk} \approx c F_{Cas} h_1 h_2 / r^2$  where  $F_{Cas}$  is the CF for perfectly conducting planes and  $c$  is a dimensionless constant. This estimation can be relevant for modern experiments on the CE. For instance, in [4] there were results obtained for parallel metallic surfaces where width of layer was about  $h \approx 50$  nm and typical distance  $r$  between surfaces was  $0.5 \mu m \leq r \leq 3 \mu m$ . In that case  $3 \times 10^{-4} \leq (h/r)^2 \leq 10^{-2}$ . In [4] authors have fitted the CF between chromium films with function  $C_{Cas}/r^4$ . They claim that the value of  $C_{Cas}$  coincides with known Casimir result within a 15% accuracy. It means that bulk force can be neglected, and only surface effects are essential. In our model the values  $a > 4.8$  of defect coupling parameter  $a$  are in good agreement with results of [4].

Now we study the scattering of classical electromagnetic wave on plane defect and effects generated by coupling of plane film with a given classical 4-current.

#### 4. Interaction of film with classical current and electromagnetic waves

The scattering problem is described in our approach by a homogeneous classical equation  $K_{2P}^{\mu\nu} A_\nu = 0$  of simplified model with  $a_1 = a$ ,  $a_2 = 0$ . It has a solution in the form of a plane wave. If one defines transmission (reflection) coefficient as a ratio  $K_t = U_t/U_{in}$ , ( $K_r = U_r/U_{in}$ ) of transmitted wave energy  $U_t$  (reflected wave energy  $U_r$ ) to incident wave energy  $U_{in}$ , then direct calculations give the following result:  $K_t = (1 + a^2)^{-1}$ ,  $K_r = a^2(1 + a^2)^{-1}$ . We note the following features of reflection and transmission coefficients. In the limit of infinitely large defect coupling these coefficients coincide with coefficients for a perfectly conducting plane. The reflection and transmission coefficients do not depend on the incidence angle.

The classical charge and the wire with current near defect plane are modeled by appropriately chosen 4-current  $J$  in (3). The mean vector potential  $\mathcal{A}_\mu$  generated by  $J$  and the plane  $x_3 = 0$ , with  $a_1 = a$  can be calculated as

$$\mathcal{A}^\mu = -i \frac{\delta G(J)}{\delta J_\mu} \Big|_{a_1=a, a_2=0} = i D_{2P}^{\mu\nu} J_\nu \Big|_{a_1=a, a_2=0}. \quad (5)$$

Using notations  $\mathcal{F}_{ik} = \partial_i \mathcal{A}_k - \partial_k \mathcal{A}_i$ , one can present electric and magnetic fields as  $\vec{E} = (\mathcal{F}_{01}, \mathcal{F}_{02}, \mathcal{F}_{03})$ ,  $\vec{H} = (\mathcal{F}_{23}, \mathcal{F}_{31}, \mathcal{F}_{12})$ . For charge  $e$  at the point  $(x_1, x_2, x_3) = (0, 0, l)$ ,  $l > 0$  the corresponding classical 4-current is

$$J_\mu(x) = 4\pi e \delta(x_1) \delta(x_2) \delta(x_3 - l) \delta_{0\mu}$$

In virtue of (5) the mean vector potential  $\mathcal{A}^\mu(x)$  is independent on  $x_0$  and the electric field in considered system is defined by potential

$$\mathcal{A}_0(x_1, x_2, x_3) = \frac{e}{\rho_-} - \frac{a^2}{a^2 + 1} \frac{e}{\rho_+}.$$

where  $\rho_+ \equiv \sqrt{x_1^2 + x_2^2 + (|x_3| + l)^2}$ ,  $\rho_- \equiv \sqrt{x_1^2 + x_2^2 + (x_3 - l)^2}$ . The electric field  $\vec{E} = (E_1, E_2, E_3)$  is of the form

$$E_1 = \frac{ex_1}{\rho_-^3} - \frac{a^2}{a^2 + 1} \frac{ex_1}{\rho_+^3}, \quad E_2 = \frac{ex_2}{\rho_-^3} - \frac{a^2}{a^2 + 1} \frac{ex_2}{\rho_+^3}, \quad E_3 = \frac{e(x_3 - l)}{\rho_-^3} - \frac{a^2 \epsilon(x_3)}{a^2 + 1} \frac{e(|x_3| + l)}{\rho_+^3}.$$

Here,  $\epsilon(x_3) \equiv x_3/|x_3|$ . We see that for  $x_3 > 0$  the field  $\vec{E}$  coincides with field generated in usual classical electrostatic by charge  $e$  placed on distance  $l$  from infinitely thick slab with dielectric constant  $\epsilon = 2a^2 + 1$ .

Because  $\mathcal{A}^\mu(x) \neq 0$  for  $\mu = 1, 2, 3$ , the defect generate also a magnetic field  $\vec{H} = (H_1, H_2, H_3)$ :

$$H_1 = \frac{e a x_1}{(a^2 + 1) \rho_+^3}, \quad H_2 = \frac{e a x_2}{(a^2 + 1) \rho_+^3}, \quad H_3 = \frac{e a (|x_3| + l)}{(a^2 + 1) \rho_+^3}.$$

It is an anomalous field which doesn't arise in classical electrostatics. Its direction depends on sign of  $a$ . In similar one can calculate the fields generated by interaction of the film and charged plane  $x_3 = l$ , presented by the classical current

$$J_\mu(x) = 4\pi \sigma \delta(x_3 - l) \delta_{0\mu}.$$

Here  $\sigma$  is the charge density. In this case it holds:

$$E_1 = E_2 = 0 = H_1 = H_2 = 0, \quad E_3 = 2\pi\sigma \left( \epsilon(x_3 - l) - \epsilon(x_3) \frac{a^2}{a^2 + 1} \right), \quad H_3 = 2\pi\sigma \frac{a}{a^2 + 1}.$$

Thus, in considered system there is only one dependent on  $l$  component of fields  $\vec{E}$ ,  $\vec{H}$ . It is  $E_3$ . For  $l \rightarrow \mp\infty$

$$E_3 = 2\pi\sigma \left( \pm 1 - \frac{\epsilon(x_3)a^2}{a^2 + 1} \right),$$

and for  $l = 0$

$$E_3 = \frac{2\pi\sigma\epsilon(x_3)}{a^2 + 1}.$$

It is important, to note that anomalous fields arise because the space parity is broken by the action (4), and they are generated in (5) by the  $L\mathcal{P}_2$ - term of propagator  $D_{2P}$ .

A current with density  $j$  flowing in the wire along the  $x_1$ -axis is modeled by

$$J_\mu(x) = 4\pi j \delta(x_3 - l) \delta(x_2) \delta_{\mu 1}$$

For magnetic field from (5) one obtains in region  $x_3 > 0$  the usual results of classical electrodynamics for the current parallel to infinitely thick slab with permeability  $\mu = (2a^2 + 1)^{-1}$ . There is also an anomalous electric field  $\vec{E} = (0, E_2, E_3)$ :

$$E_2 = \frac{2ja}{a^2 + 1} \frac{x_2}{\tau^2}, \quad E_3 = \frac{2ja}{a^2 + 1} \frac{|x_3| + l}{\tau^2}$$

where  $\tau = (x_2^2 + (|x_3| + l)^2)^{\frac{1}{2}}$ . Comparing formulae  $\epsilon = 2a^2 + 1$  and  $\mu = (2a^2 + 1)^{-1}$  for parameter  $a$  we obtain the relation  $\epsilon\mu = 1$ . It holds for material of thick slab interaction of which with point charge and current in classical electrodynamics was compared with results for thin film of our model. The speed of light in this hypothetical material is equal to one in the vacuum. From the physical point of view, it could be expected, because interaction of film with photon field is a surface effect which can not generate the bulk phenomena like decreasing the speed of light in the considered slab. With this arguments it seems to be not surprisingly that the reflection coefficient of electromagnetic wave in our model is independent from the incidence angle, since by  $\epsilon\mu = 1$  it holds for Fresnel formulas too.

The relation  $\epsilon\mu = 1$  is not new in the context of the Casimir theory. It was first introduced by Brevik and Kolbenstvedt [15] who calculated the Casimir surface force density on the sphere. Only on this condition a contact term turn out to be zero [15]. It has been investigated in a number of subsequent papers. In our approach this condition arises naturally because we have only one parameter  $a$  that must describe both magnetostatic and electrostatic properties of the film.

The essential property of interaction of films with classical charge and current is the appearance of anomalous fields. This fields are suppressed in respect of usual ones by factor  $a^{-1}$  and they vanish in case of perfectly conducting plane. Magnetoelectric (ME) films are good candidates to detect anomalous fields and non ideal CE. The generic example of ME crystals is  $Cr_2O_3$  [16]. It is important to note that for ME films the Lifshitz theory of CE is not relevant but they can be studied in our approach.

## 5. Conclusion

The main results of our study on the CE for thin films in the QED are the following. We have shown that if the CF holds true for thin material film, then an interaction of this film with the QED fields can be modeled by photodynamic with the defect action (1) obtained by most general assumptions consistent with locality, gauge invariance and renormalizability of model. Thus, basic principles of QED were essential in our studies of the CE. These principles make it possible to expose new peculiarities of the physics of macroscopic objects in QED and must be taken into account for construction of the models. For plane films we have demonstrated that the CF is not universal and depends on properties of the material represented by the parameter  $a$ . For  $a \rightarrow \infty$  one can obtain the CF for ideal conducting planes. In this case the model coincides with photodynamic considered in [17] with boundary condition  $\epsilon^{ijk}F_{jk} = 0$  ( $i = 0, 1, 2$ ) on orthogonal to the  $x_3$ -axis planes. For sufficiently small  $a$  the CF appears to be repulsive. Interaction of plane films with charges and currents generate anomalous magnetic and electric fields which do not arise in classical electrodynamics. The ME materials could be used for observation of phenomena predicted by our model. We hope that the obtained theoretical results can be proven by modern experimental methods.

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